Can Inflating Braneworlds be Stabilized?

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Talk Outline:

Braneworld model:

- warped geometry background
- bulk scalar fields and phase portraits

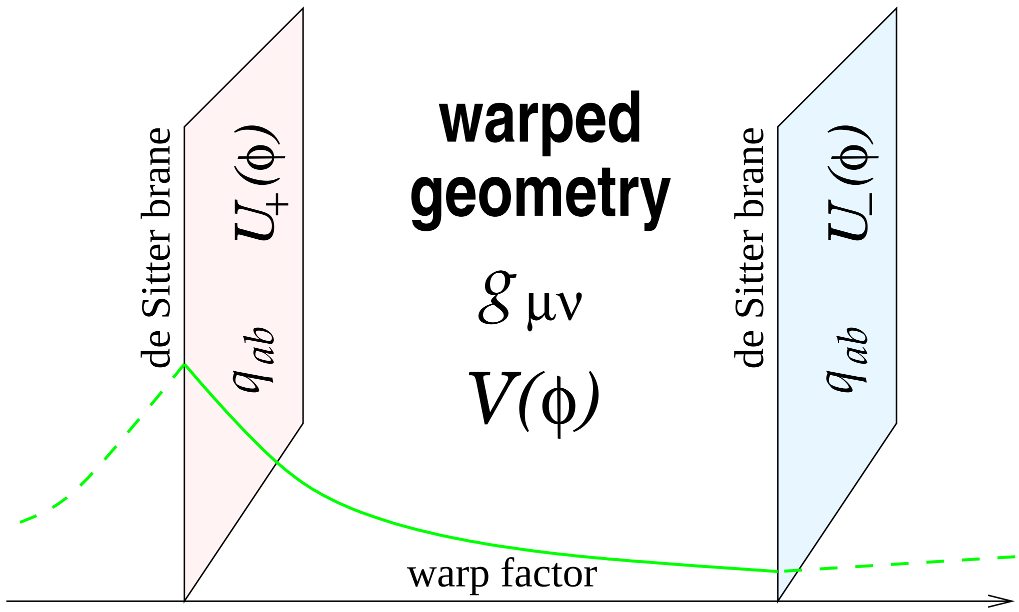
(In)Stability of inflating braneworlds:

- scalar bulk perturbations
- brane embedding and boundary conditions
- KK mass spectrum and the lowest mode
- the fate of the unstable braneworld

Braneworld Geometry

- "Warped" braneworld metric:

$$ds^2 = a^2(w)\left[dw^2 \overbrace{- dt^2 + e^{2Ht} d\vec{x}^2}^{\text{de Sitter slices}}\right]$$



- Background field equations and boundary conditions:

bulk equations

$$\varphi'' + 3\frac{a'}{a}\varphi' - a^2 V' = 0$$

$$\frac{a''}{a} = 2\,\frac{a'^2}{a^2} - H^2 - \frac{\varphi'^2}{3}$$

$$6\left(\frac{a'^2}{a^2} - H^2\right) = \frac{\varphi'^2}{2} - a^2 V$$

brane BCs

$$\frac{\varphi'}{a} = \pm\frac{U'}{2}$$

$$\frac{a'}{a^2} = \mp\frac{U}{6}$$

Scalar Field Action

- Einstein-Hilbert action with scalar field:

$$S = M_5^3 \int \sqrt{-g}\, d^5x \,\{R - (\nabla\varphi)^2 - 2V(\varphi)\}$$
$$-2M_5^3 \sum \int \sqrt{-q}\, d^4x \,\{[\mathcal{K}] + U(\varphi)\}$$

- Bulk Einstein and scalar field equations:

$$\boxed{R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} = T_{\mu\nu}, \quad \Box\varphi = \frac{\partial V}{\partial\varphi}}$$

- Bulk scalar field stress-energy tensor:

$$T_{\mu\nu} = \varphi_{,\mu}\varphi_{,\nu} + \left\{-\frac{1}{2}(\nabla\varphi)^2 - V(\varphi)\right\} g_{\mu\nu}$$

- Induced metric and extrinsic curvature:

$$q_{ab} = e^{\mu}_{(a)} e^{\nu}_{(b)} g_{\mu\nu}, \quad \mathcal{K}_{ab} = e^{\mu}_{(a)} e^{\nu}_{(b)} \nabla_\mu n_\nu$$

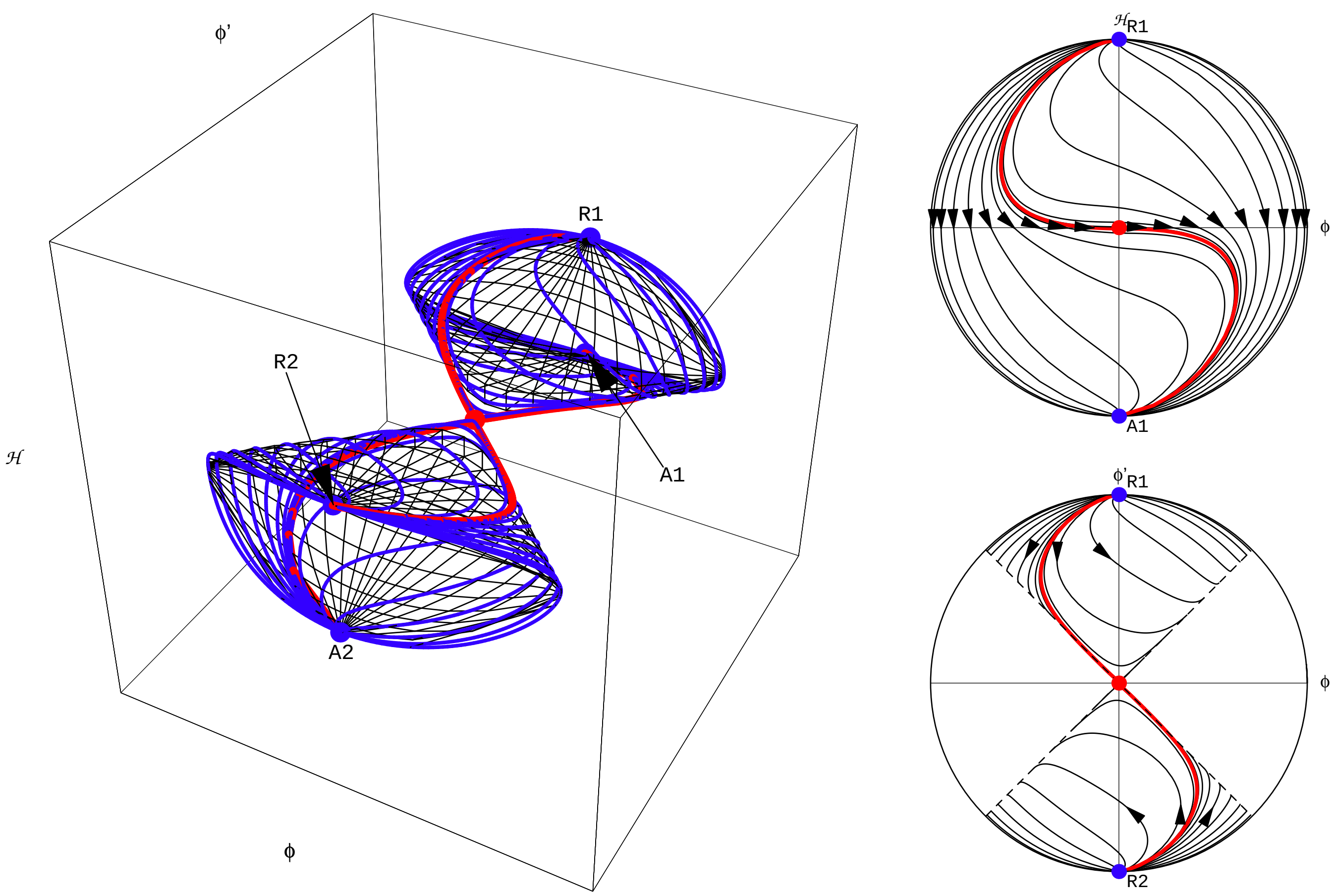
- Junction conditions at the branes:

$$\boxed{[\mathcal{K}_{ab} - \mathcal{K} q_{ab}] = U(\varphi) q_{ab}, \quad [n \cdot \nabla\varphi] = \frac{\partial U}{\partial\varphi}}$$

Phase Portrait of Scalar Field Equations

- Four dynamical variables: $\left\{a, \varphi, \frac{a'}{a^2}, \frac{\varphi'}{a}\right\}$
- Phase space dimension can be reduced to three
- $H = 0$ trajectories form two-dimensional surface
- Boundary conditions form one-dimensional curve

Phase Portrait:



$$V(\phi) = \frac{1}{2}m^2\phi^2 + \Lambda, \quad \Lambda = 0$$

Scalar Bulk Perturbations

- Scalar metric perturbations: $\left[\Psi = -\frac{\Phi}{2}\right]$

$$ds^2 = a(w)^2 \left[(1+2\Phi)dw^2 + (1+2\Psi)ds_4^2\right]$$

- Mode decomposition:

$$\Phi(x^A) = \sum_m \Phi_m(w) Q_m(t, \vec{x})$$

$$^4\Box Q_m = m^2 Q_m$$

- Linearized Einstein equations:

$$\begin{aligned} (a^2\Phi)' &= \frac{2}{3}a^2\varphi'\,\delta\varphi \\ \left(\frac{a}{\varphi'}\,\delta\varphi\right)' &= \left(1 - \frac{3}{2}\frac{m^2+4H^2}{\varphi'^2}\right)a\Phi \end{aligned}$$

- Can be combined into Schrödinger form:

$$u_m'' + \left(m^2 + 4H^2 - V_{\text{eff}}(w)\right)u_m = 0$$

$$u_m = \sqrt{\tfrac{3}{2}}\,\frac{a^{3/2}}{\varphi'}\,\Phi_m, \quad V_{\text{eff}} = \frac{z''}{z} + \frac{2}{3}\varphi'^2, \quad z = \left(\tfrac{2}{3}a\varphi'^2\right)^{-\frac{1}{2}}$$

Brane Embedding and Boundary Conditions

- Holonomic basis and unit normal:

$$e^{\mu}_{(a)} \equiv \frac{\partial x^{\mu}}{\partial x^{a}} = \left(\xi_{,a}, \delta^{\mu}_{a}\right), \quad n_{\mu} = a\left(1 + \Phi, -\xi_{,a}\delta^{a}_{\mu}\right)$$

- Induced metric is conformally flat:

$$d\sigma^2 = a^2(1 - \Phi)\left[-dt^2 + e^{2Ht}d\vec{x}^2\right]$$

- Extrinsic curvature: $\mathcal{K}_{ab} = e^{\mu}_{(a)} e^{\nu}_{(b)} n_{\mu;\nu}$

$$\mathcal{K} = 4\frac{a'}{a^2} - 2\frac{(a^2\Phi)'}{a^3} - \frac{{}^4\Box\xi}{a}$$

- Junction conditions across the brane:

background

$$\frac{a'}{a^2} = \mp\frac{U}{6}, \qquad \frac{\varphi'}{a} = \pm\frac{U'}{2},$$

perturbation

$$(a^2\Phi)'\Big|_{w_\pm} = \pm\frac{1}{3}\,U' a^3\,\delta\varphi\Big|_{w_\pm}$$

$$(\delta\varphi' - \varphi'\Phi)\Big|_{w_\pm} = \pm\frac{1}{2}\,U'' a\,\delta\varphi\Big|_{w_\pm}$$

- Use bulk equations to rewrite as:

$$\left(\frac{a}{\varphi'}\,\delta\varphi\right)\Bigg|_{w_\pm} = \frac{3}{2}\,\frac{m^2 + 4H^2}{a\varphi'^2}\,\frac{a^2\Phi}{\frac{a^2V'}{\varphi'} - 4\frac{a'}{a} \mp aU''_{\pm}}\Bigg|_{w_\pm}$$

- Rigid stabilization limit: U'' large

$$\boxed{\delta\varphi\Big|_{w_\pm} = 0}$$

KK Mass Spectrum and the Lowest Mode

- Self-adjoint boundary value problem: $Y = a^2\Phi$

$$\mathcal{D}Y \equiv -(gY')' + fY = \lambda g Y,$$
$$Y'(w_\pm) = 0$$

$$f = \frac{1}{a}, \qquad g = \frac{1}{\frac{2}{3}a\varphi'^2}, \qquad \lambda = m^2 + 4H^2$$

- Rigorous lower and upper bounds:

$$0 \le \lambda_1 \le \frac{\int F\mathcal{D}F\, dw}{\int gF^2\, dw}$$

- Taking a trial function $F = 1$:

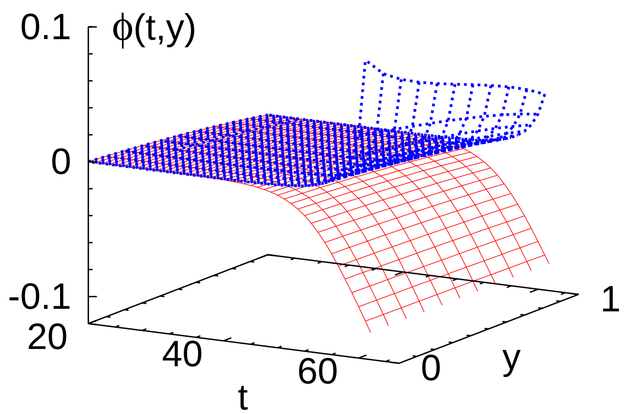
$$0 \le \lambda_1 \le \frac{\int f\, dw}{\int g\, dw}$$

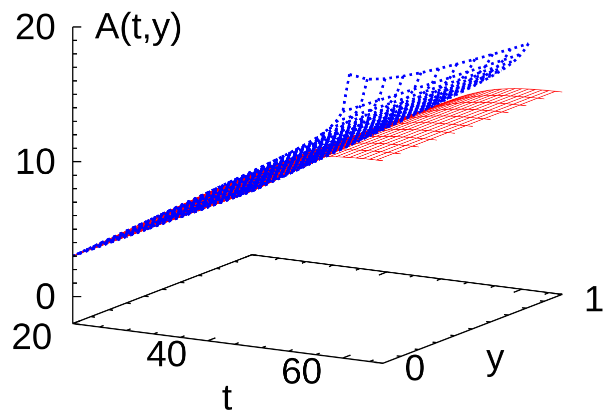
- Lowest mass eigenvalue:

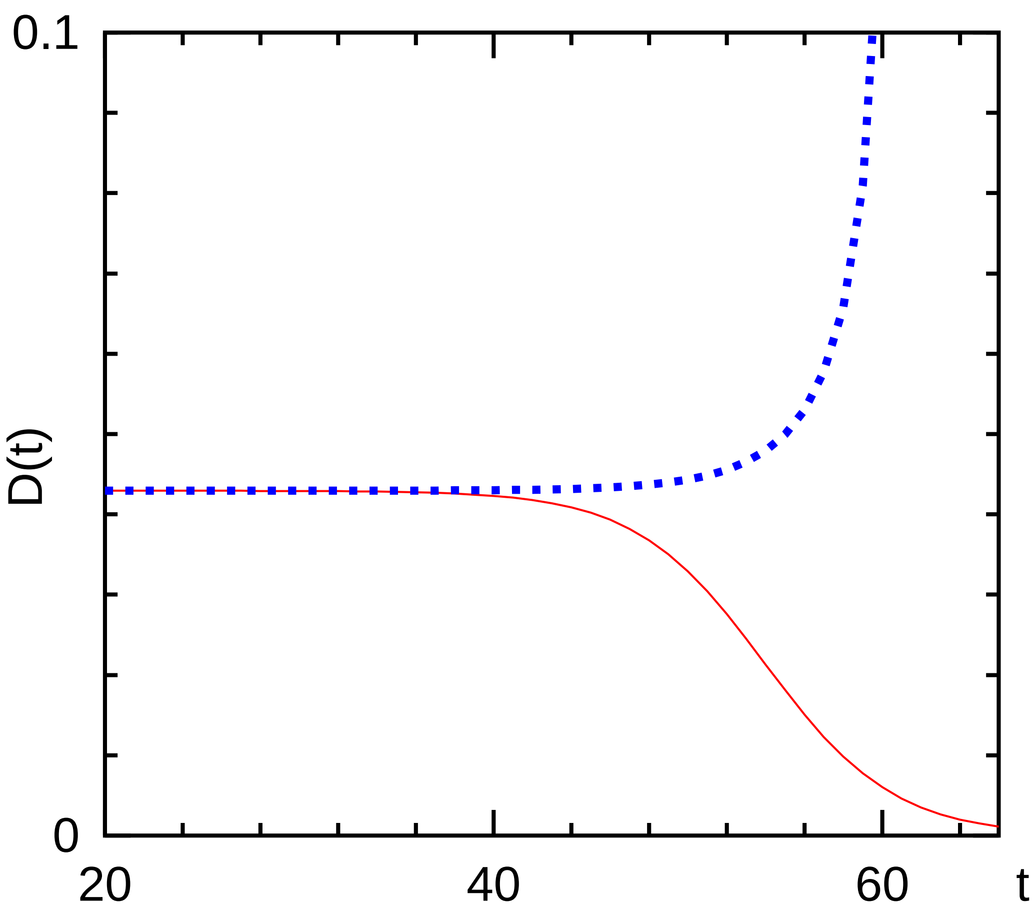
$$\boxed{-4H^2 \le m^2 \le -4H^2 + \overbrace{\frac{2}{3}\frac{\int \frac{dw}{a}}{\int \frac{dw}{a\varphi'^2}}}^{m_0^2}}$$

Negative mass means instability!

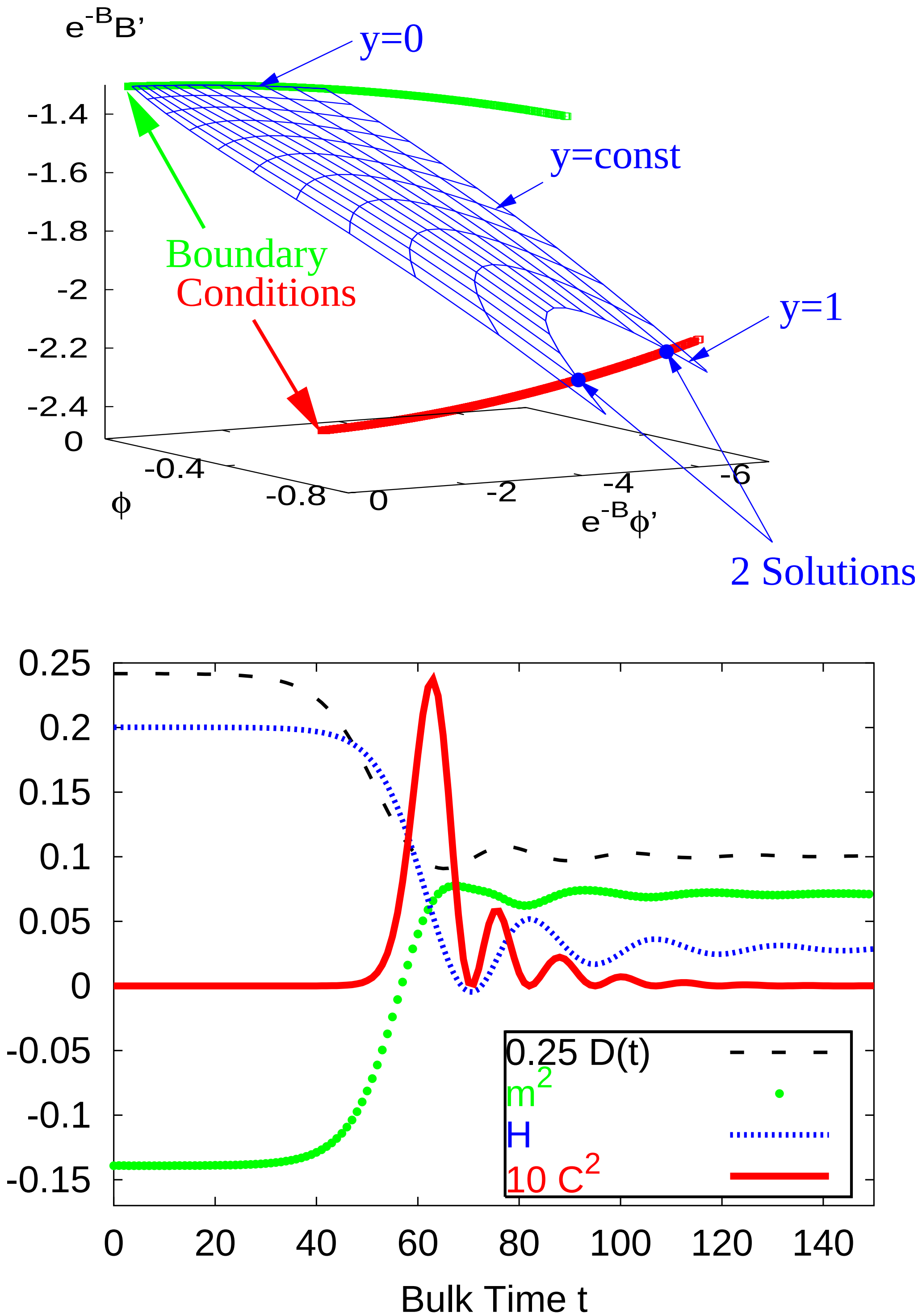
The Fate of the Unstable Braneworld







More Interesting Things Can Happen



Transition between two static brane configurations

Summary

Inflating braneworlds are hard to stabilize!

➠ Gravitational wave modes:

- The lightest mode has $m = 0$ (4D graviton)
- Mass gap in KK spectrum $m \geq \sqrt{3/2}\, H$
- Massive KK graviton modes are not generated!

➠ Scalar (“radion”) modes:

- The lightest mode has $m^2 = -4H^2 + m_0^2(H)$
- Strong tachyonic instability regardless of $U(\varphi)$
- Light radion produces cosmological fluctuations

➠ Non-linear braneworld dynamics:

- Decompactification or brane collision
- Transition between two static brane configurations
- More interesting dynamics?